# Pink Kangaroo



*a member of the Association Kangourou sans Frontières*

## Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

Enquiries about the Pink Kangaroo should be sent to:

*challenges@ukmt.org.uk*

www.ukmt.org.uk

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | D | A | E | E | D | B | C | D | C | D | E | D | B | C | B | C | C | A | D | A | C | D | B | A |

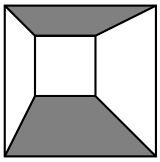
**1.** Today is Thursday. What day will it be in 2023 days' time?

A Tuesday  B Wednesday  C Thursday  D Friday  E Saturday

SOLUTION **C**

Because 2023 is divisible by 7, it will be on the same day of the week, namely Thursday.

**2.** A large square of side-length 10 cm contains a smaller square of side-length 4 cm, as shown in the diagram. The corresponding sides of the two squares are parallel. What percentage of the area of the large square is shaded?



A 25%  B 30%  C 40%  D 42%  E 45%

SOLUTION **D**

The length of the big square is 10 cm and of the smaller is 4 cm. The total height of the shaded regions is 6. Hence the total area of the two trapezoids combined is $\frac{10+4}{2} \times 6 = 42\,\text{cm}^2$. Since the total area is $100\,\text{cm}^2$ this is 42 %.

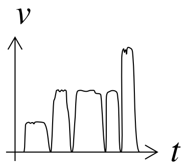
**3.** A wooden fence consists of a series of vertical planks, each joined to the next vertical plank by four horizontal planks. The first and last planks in the fence are vertical. Which of the following could be the total number of planks in the fence?

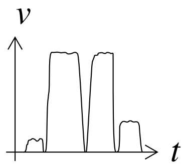
A 96  B 97  C 98  D 99  E 100

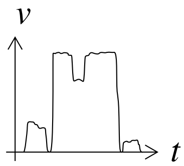
SOLUTION **A**

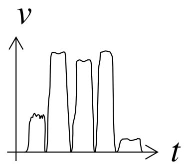
The first panel of this fence consists of 6 planks (2 vertical and 4 horizontal); every additional panel consists of 5 planks (another vertical and 4 horizontal). So the number must be some number of the form $5x + 1$ for $x \geq 1$. The only such number listed is 96.

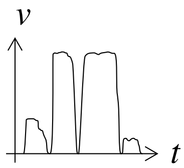
**4.** Mary had to run to catch the train, got off two stops later and then walked to school. Which of the following speed-time graphs would best represent her journey?

A 


B 


C 


D 


E 


Solution **E**

The order of speeds needs to be train > running > walk. This rules out $A$ and $B$. The train section of the trip needs to have just a single intermediate stop in the middle of the trip. However $C$ shows no intermediate stop, just a slowing, and $D$ shows two intermediate stops. This leaves $E$ as the only feasible solution.

**5.** Alec has won 49% of the 200 games of chess he has played. He would like to have won exactly 50% of his games. What is the smallest number of extra games he needs to play?

A 0 B 1 C 2 D 3 E 4

Solution **E**

Winning 49% of 200 games means winning 98 games and not winning 102. For Alec to have equal number of wins and not-wins he needs to win $102 - 98$ games, that is 4 games. This will mean that he will have 102 wins out of 204 making a 50 % rate of success.

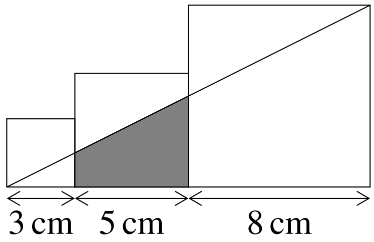
**6.** Lucy is trying to save water. She has reduced the time she spends in her shower by one quarter. Also, she has lowered the water pressure to reduce the rate the water comes out of the shower head by a quarter. By what fraction has Lucy reduced the total amount of water she uses when she showers?

A $\frac{3}{8}$ B $\frac{1}{16}$ C $\frac{5}{12}$ D $\frac{7}{16}$ E $\frac{9}{16}$

Solution **D**

In total Lucy uses $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$ of the water volume that she used before. So she has reduced the total amount of water by $\frac{7}{16}$.

**7.** The diagram shows three squares of side-length 3 cm, 5 cm and 8 cm. What is the area, in $cm^2$, of the shaded trapezium?




A 13 B $\frac{55}{4}$ C $\frac{61}{4}$ D $\frac{65}{4}$ E $\frac{69}{4}$

Solution **B**

Let the lengths of the vertical sides of the shaded trapezium be $p$ and $q$. Using similar triangles, $\frac{p}{3} = \frac{q}{3+5} = \frac{8}{3+5+8}$. Hence $p = \frac{3}{2}$ and $q = 4$.

Therefore the area of the trapezium is $\frac{1}{2} \times (\frac{3}{2} + 4) \times 5$ that is $\frac{55}{4}$ $cm^2$.

**8.** A wire of length 95 m is cut into three pieces such that the length of each piece is 50 % greater than the previous piece. What is the length of the largest piece?

A 36 m B 42 m C 45 m D 46 m E 48 m

Solution **C**

Let $v$ be the length of the smallest piece. Then the middle piece has length $\frac{3}{2}v$ and the largest has length $\frac{3}{2} \times \frac{3}{2}v = \frac{9}{4}v$. So the total length is $\frac{9}{4}v + \frac{3}{2}v + v = \frac{19}{4}v$ and hence $v = 20$ m and the longest piece has length $\frac{9}{4}v$, that is 45 m.

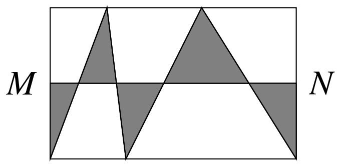
**9.** The ages of a family of five sum to 80. The two youngest are 6 and 8. What was the sum of the ages of the family seven years ago?

A 35 B 36 C 45 D 46 E 66

Solution **D**

Seven years ago, the youngest child had not been born and so did not contribute to the sum of their ages. The sum of the four eldest members of the family is 74. Therefore, seven years ago, the sum of their ages was $74 - 28$, that is 46.

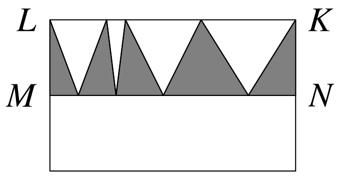
**10.** Points $M$ and $N$ are the midpoints of two sides of the rectangle, shown in the diagram. What fraction of the rectangle's area is shaded?



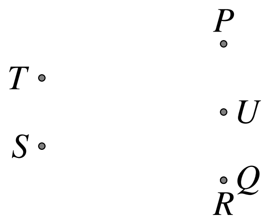

A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E $\frac{1}{2}$

Solution **C**

The diagram here shows all of the lower triangles reflected to be above the line $MN$. This makes clear that the total area of all of the shaded triangles is $\frac{1}{2} \times MN \times ML$. This area is half that of the rectangle $MNKL$, and so equals $\frac{1}{4}$ of the original rectangle.




**11.** The Pentagon $PQRST$ is divided into four triangles with equal perimeters. The triangle $PQR$ is equilateral. $PTU$, $SUT$ and $RSU$ are congruent isosceles triangles. What is the ratio of the perimeter of the pentagon $PQRST$ to the perimeter of the triangle $PQR$?




A 2 : 1 B 3 : 2 C 4 : 3 D 5 : 3 E 5 : 2

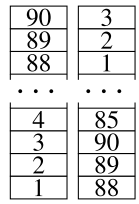
Solution **D**

Let $2a$ be the side of equilateral triangle $PQR$. Then $PQR$ has a perimeter of $6a$ and $PU$ has a length $a$.

Therefore, in order for the perimeter of triangle $PTU$ to be $6a$, $PT$ must be $\frac{5}{2}a$.

Since the isosceles triangles are congruent $ST = PU$ and $RS = PT$. So the perimeter of the pentagon $PQRST$ is $2a + 2a + \frac{5}{2}a + a + \frac{5}{2}a = 10a$. Therefore the ratio wanted is $10a : 6a$, that is 5 : 3.

**12.** On the table there is a tower made of blocks numbered from 1 to 90, as shown on the left of the diagram. Yett takes blocks from the top of the tower, three at a time, to build a new tower, as shown on the right of the diagram. How many blocks will be between blocks 39 and 40 when he has finished building the new tower?




A 0 B 1 C 2 D 3 E 4

Solution **E**

If $n$ is divisible by 3, then block $n$ will be be moved with the two blocks underneath it, $n-1$ and $n-2$.

Since 42 is divisible by 3, block 42 will be moved with blocks number 41 and 40 underneath block 42. So now our new tower from top down looks like: $42, 41, 40, 45, 44, \ldots$.

Yett would then next move the three blocks that are on the top of what remains of the original tower, namely 39, 38 and 37. Adding these to our previous set gives the order as: $\mathbf{39}, 38, 37, 42, 41, \mathbf{40}, 45, 44, \ldots$.

It follows that between blocks with numbers 39 and 40 there will be 4 blocks.

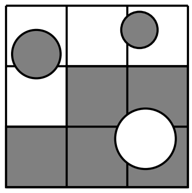
**13.** We will call a two-digit number **power-less** if neither of its digits can be written as an integer to a power greater than 1. For example, 53 is power-less, but 54 is *not* power-less since $4 = 2^2$. Which of the following is a common divisor of the smallest and the largest power-less numbers?

A 3 B 5 C 7 D 11 E 13

Solution **D**

The digits of a power-less number can be only 2, 3, 5, 6, and 7. The smallest two-digit number is 22 and the largest one is 77, and they have a common divisor of 11.

**14.** A square of side-length 30 cm is divided into nine smaller identical squares. The large square contains three circles with radii 5 cm (bottom right), 4 cm (top left) and 3 cm (top right), as shown. What is the total area of the shaded part?



A $400\,\text{cm}^2$ B $500\,\text{cm}^2$
C $(400 + 50\pi)\,\text{cm}^2$ D $(500 - 25\pi)\,\text{cm}^2$
E $(500 + 25\pi)\,\text{cm}^2$

SOLUTION **B**

The sum of the areas of the two smaller grey circles is the same as that of the white circle since $(3\pi)^2 + (4\pi)^2 = (5\pi)^2$. This means that the shaded area is the same as the area of the five of the smaller squares. Each of the smaller squares has and area of $10\,\text{cm} \times 10\,\text{cm} = 100\,\text{m}^2$. The shaded area is then $5 \times 100\,\text{cm}^2 = 500\,\text{cm}^2$.

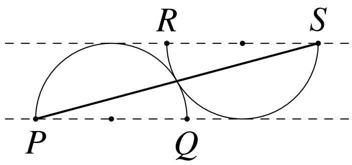
**15.** Jenny calculates the average of five different prime numbers. Her answer is an integer. What is the smallest possible integer she could have obtained?

A 2 B 5 C 6 D 12 E 30

SOLUTION **C**

For the average to be an integer the sum of 5 primes must be divisible by 5. The sum of the first 5 primes is $2 + 3 + 5 + 7 + 11 = 28$ which is not a multiple of 5. However, the next smallest sum of five primes is $2 + 3 + 5 + 7 + 13 = 30$ and the average of these five is 6.

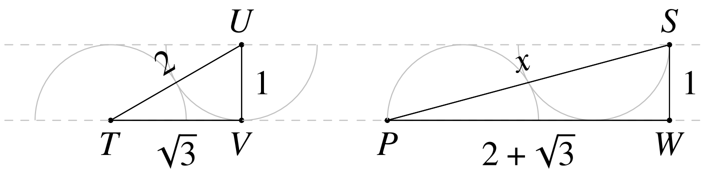
**16.** The figure shows two touching semicircles of radius 1, with parallel diameters $PQ$ and $RS$. What is the square of the distance $PS$?




A 16 B $8 + 4\sqrt{3}$ C 12
D 9 E $5 + 2\sqrt{3}$

SOLUTION **B**

Let $T$ and $U$ be the centres of the two semicircles and $V$, $W$ be where the perpendiculars from $U$ and $S$ meet $PQ$. Then $TU$ has length 2, and $UV$ and $SW$ have length 1. Using Pythagoras' theorem $TV$ is $\sqrt{3}$.




This means that $PW$ is $2 + \sqrt{3}$ as it is $TV$ + 2 radii. So $SP^2 = (2 + \sqrt{3})^2 + 1^2$ which is $8 + 4\sqrt{3}$.

**17.** Ireena is extending a sequence of numbers with the following rule. The next term in the sequence is the smallest non-negative integer that is different from each of the four preceding terms. She then repeats this process over and over again. For instance, if Ireena was to start with the sequence 7, 3, 1, 8 then the fifth and sixth terms of the sequence would be 0 and 2 respectively.

Ireena starts with the sequence

$$2,\ 0,\ 2,\ 3.$$

What is the 2023rd number in this sequence?

A 0 B 1 C 2 D 3 E 4

Solution **C**

We easily find successive numbers: 2, 0, 2, 3, 1, 4, 0, 2, 3, 1, 4, 0, 2, 3, 1, 4, . . . .

We note that, after the first term, the sequence consists of a cycle of 5 numbers (0, 2, 3, 1, 4) that then repeats.

The number we seek is the 2023rd term in the original sequence which is the 2022nd number in the repetition of the cycle. This will be the same as the second number in the cycle, namely 2.

**18.** A group of students took a test which consisted of 3 questions. We know that 90 % answered the first question correctly, 80 % answered the second question correctly and 70 % answered the third question correctly. What is the smallest possible percentage of students who answered all three questions correctly?

A 30 % B 35 % C 40 % D 50 % E 70 %

Solution **C**

There are a group of 10 % who failed to answer correctly the first question, 20 % for the second and 30 % for the third.

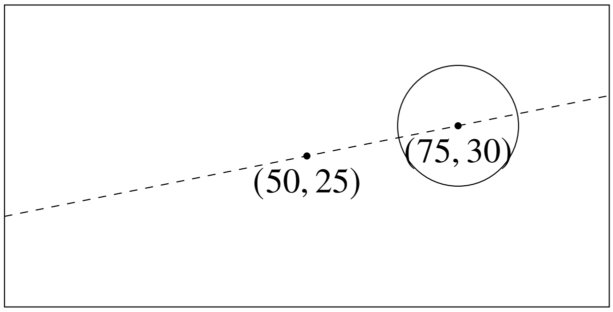
The number who failed on one or more questions will be largest if these groups do not overlap; and in that case they will form 60 % of the whole group. So the smallest possible percentage of students who got all correct is 40 %.

**19.** A rectangle with vertices $(0, 0)$, $(100, 0)$, $(100, 50)$ and $(0, 50)$ has a circle with centre $(75, 30)$ and radius 10 cut out of it. What is the slope of the line through the point $(75, 30)$ which divides the remaining area of the rectangle into two shapes of equal area?

A $\frac{1}{5}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{2}{5}$ E $\frac{2}{3}$

Solution **A**

Any line that bisects the area of the circle must pass through its centre. Similarly, any line that bisects the area of the rectangle must pass through the centre of the rectangle.




So, since the circle lies wholly within the rectangle, the line passing through the centre of the rectangle $(50, 25)$ and the centre of the circle $(75, 30)$ must cut the remaining area in half, as a semicircle is cut out of each side. The slope of that line is

$$\frac{30-25}{75-50} = \frac{1}{5}.$$

**20.** Eva chooses a three-digit positive number and from it she subtracts the sum of its three digits. She finds that the answer is a three-digit number in which all three digits are the same. How many different starting numbers could Eva have chosen?

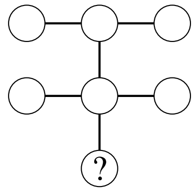
A 2 B 5 C 10 D 20 E 30

Solution **D**

Let the three-digit number be '$abc$'. Then, the difference between the number and the sum of its digits is $100a + 10b + c - (a + b + c) = 99a + 9b$. This difference is divisible by 9. Therefore the result can only be 333 or 666 or 999, and it is easy to see that 999 is not possible.

However, 333 corresponds to $11a + b = 37$ so $a = 3$, $b = 4$ and we get the numbers $340, 341, 342, \ldots, 349$. Similarly 666 corresponds to $11a + b = 74$ so that $a = 6$, $b = 8$ and we get the numbers $680, 681, 682, \ldots, 689$. Therefore there are 20 different numbers with this property.

**21.** Seven different single-digit numbers are written in the circles of the diagram shown with one number in each circle. The product of the three numbers in each of the three lines of three numbers is the same. Which number is written in the circle containing the question mark?




A 2  B 3  C 4  D 6  E 8

Solution **A**

Consider the number 5. If it were used then, wherever it was placed, there would be at least one line to which it belongs and at least one to which it doesn't. The product of those lines could not be equal. The same argument applies to 0 and to 7.

So the seven entries must be 1, 2, 3, 4, 6, 8 and 9 and their product is $1\times2\times3\times4\times6\times8\times9 = 2^7\times3^4$.

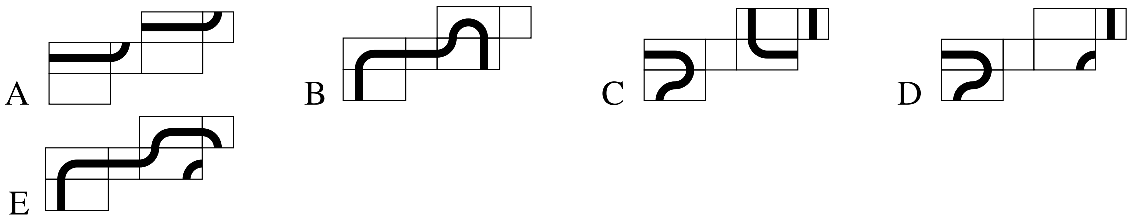
The product of the numbers in the two horizontal rows must be a perfect square (the square of the common product). So the bottom digit must either be 2 or $2^3 = 8$ to account for the difference.

But it cannot be 8 because then the product of each row would be $\sqrt{2^4 \times 3^4}$, which is 36. But this is not a multiple of 8.

So the number in the circle is 2.

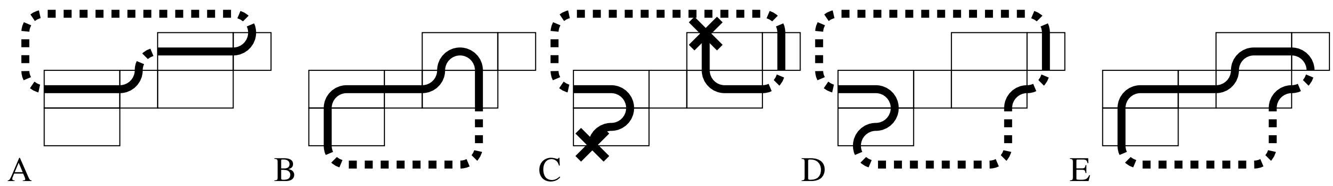
This is possible, with the first row 1, 9, 8, second row 3, 4, 6 and column 9, 4, 2, with common product 72.

**22.** Lancelot has drawn a closed path on a cuboid and unfolded it into a net. Which of the nets shown could *not* be the net of Lancelot's cuboid?




Solution **C**

The nets given can be folded into a solid in only one way. We can continue the path along the edges that would be joined. Connections shown by dashed lines show which pieces are glued together (i.e. belong to to the same path).




Following the path one can see that only C is not closed.

**23.** In how many different ways can the word **BANANA** be read from the following table by moving from one cell to another cell with which it shares an edge? Cells may be visited more than once.

| | | |
|---|---|---|
| B | A | N |
| A | N | A |
| N | A | N |

A 14 B 28 C 56 D 84 E 112

Solution **D**

**Case 1** The first N is in the 1st row, 3rd column. If the 2nd N is in the same place, then there are $2 \times 2 = 4$ possibilities. If the 2nd N is in the 2nd row, 2nd column, then there are $2 \times 4 = 8$ possibilities. If the 2nd N is in the 3rd row, 3rd column, then there are $1 \times 2 = 2$ possibilities. Altogether, there are $4 + 8 + 2 = 14$ possibilities.

**Case 2** The first N is in the 2nd row, 2nd column. If the 2nd N is in the same column, then there are $4 \times 4 = 16$ possibilities. If the 2nd N is one of the other two columns, then there are $2 \times 2 = 4$ possibilities. Altogether, there are $2(16 + 3 \times 4) = 56$ possibilities.

**Case 3** The first N is in the 1st column, 3rd row. As in the first case, 14 possibilities.

This gives a total of $56 + 14 + 14$, which is 84 possibilities.

**Alternate Method**

The numbers in the cells shown below give the number of sequences of the given letters that end in that particular cell. The numbers at the given stage are obtained from the previous stage by adding the numbers at the previous stage in all the cells from which the given cell can be reached in one move.

**B**

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

→ **BA**

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 0 | 0 | 0 |

→ **BAN**

| | | |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 2 | 0 |
| 1 | 0 | 0 |

→ **BANA**

| | | |
|---|---|---|
| 0 | 3 | 0 |
| 3 | 0 | 3 |
| 0 | 3 | 0 |

→ **BANAN**

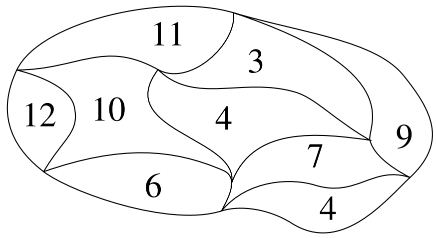
| | | |
|---|---|---|
| 0 | 0 | 6 |
| 0 | 12 | 0 |
| 6 | 0 | 6 |

→ **BANANA**

| | | |
|---|---|---|
| 0 | 18 | 0 |
| 18 | 0 | 24 |
| 0 | 24 | 0 |

The total number of ways of reading **BANANNA** is the sum of the numbers in the final diagram, that is, $18 + 18 + 24 + 24 = 84$.

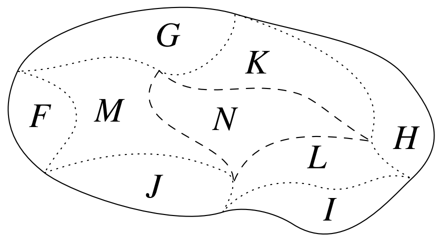
**24.** The diagram shows a map of a park. The park is divided into regions. The number inside each region gives its perimeter, in km. What is the outer perimeter of the park?



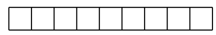

A 22 km B 26 km C 28 km
D 32 km E 34 km

Solution **B**

The sum of the perimeters of $F$, $G$, $H$, $I$ and $J$ give the length of the outside line increased by the dotted line. If we subtract the perimeters of $K$, $L$ and $M$, then we subtract the dotted line but we have now also subtracted the dashed line. So we add the dashed line to compensate. In other words the required perimeter is $(F + G + H + I + J) - (K + L + M) + N$. Here it is $42 - 20 + 4$, that is 26 km.




**25.** Vumos wants to write the integers 1 to 9 in the nine boxes shown so that the sum of the integers in any three adjacent boxes is a multiple of 3. In how many ways can he do this?



A $6 \times 6 \times 6 \times 6$
B $6 \times 6 \times 6$
C $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
D $6 \times 5 \times 4 \times 3 \times 2 \times 1$
E $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Solution **A**

Let $a, b, c, d$ be the numbers in four adjacent boxes. Then both $a + b + c$ and $b + c + d$ must be multiples of 3. Therefore $a - d$ is a multiple of 3. This applies to any entries three apart. So the numbers in the set $\{1, 4, 7\}$ must be listed three apart; and the same applies to $\{2, 5, 8\}$ and to $\{3, 6, 9\}$. This will automatically ensure that the sum of three adjacent numbers is a multiple of 3.

There are 3 choices about which of these sets go in the first, fourth and seventh boxes, 2 choices for the next set and 1 for the third. Also there are $3 \times 2 \times 1 = 6$ choices for which order the numbers in each set are given. That gives the total number of choices as $6 \times 6 \times 6 \times 6$.